1.1 Sets

Definition: a "set" is a collection of objects. The objects are called "elements."

Ex: $S = \{blue, white, red\}$.

Other notation: $S = \{x : ...\}$ means S is the set of all x such that ...

Ex: $S = \{x : x \text{ is an integer greater than } 3\} = \{4, 5, 6, \dots\}$.

Element relation: The notation $x \in A$ means that the element x belongs to the set A. The symbol $x \notin A$ means that the element x does NOT belong to the set A.

Ex: $1 \in \{1, 2, 3, 4\}$, $7 \notin \{1, 2, 3, 4\}$

Subset relation: The notation $A \subset B$ means that every element of A belongs to B as well.

Ex: Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5, 6\}$. Then $A \subset B$.

Union $\cup$: The "union" of two sets is obtained by combining the elements of both sets together, i.e. $A \cup B = \{x : x \in A \text{ OR } x \in B\}$.

Intersection $\cap$: The "intersection" of two sets is the set containing the elements common to both sets, i.e. $A \cap B = \{x : x \in A \text{ AND } x \in B\}$..

Ex: Let $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$

1. $A \cup B = \{1, 2, 3, 4\}$
2. $A \cap B = \{2, 3\}$

Empty set: The "empty set" is the set that does not contain any elements. It is denoted $\varnothing$.

Remark: $A \cup \varnothing = A$ and $A \cap \varnothing = \varnothing$ are always true.

Disjoint sets: Two sets A and B are disjoint if they do not have any elements in common; in symbols, $A \cap B = \varnothing$.

Universal set: the set that contains all the elements that we consider for a problem. Usually denoted by the letter U.

Complement: $A' = \{x : x \notin A \text{ AND } x \in U\}$ is the complement of A with respect to the universal set U.

Ex: Let $U = \{x : x \text{ is an integer}\}$ and $A = \{1, 2, 3, \dots\}$ then $A' = \{\dots, -3, -1, 0\}$.

Order of operations:

1. Parentheses
2. Complements
3. Left to right

Ex: Let $U = \{1,2,3,4,5\}$, $A = \{1,2,3\}$, and $B = \{2,4\}$, then

$$(A \cup B)' \cup B = (\{1,2,3,4\})' \cup B = \{5\} \cup B = \{2,4,5\}$$

Cartesian product: $A \times B = \{(a,b) : a \in A \text{ and } b \in B\}$.

Ex: Let $A = \{1,2,3\}$ and $B = \{2,4\}$, then

$A \times B = \{(1,2),(1,4),(2,2),(2,4),(3,2),(3,4)\}$.
$(A \cap B) \times B = \{2\} \times B = \{(2,2),(2,4)\}$.
$(A \times B) \cap (B \times B) = \{(1,2),(1,4),(2,2),(2,4),(3,2),(3,4)\} \cap \{(2,2),(2,4),(4,2),(4,4)\} = \{(2,2),(2,4))\}$.