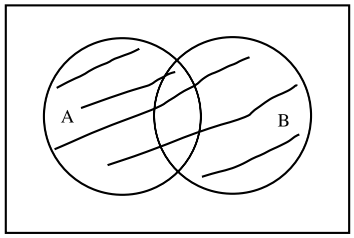
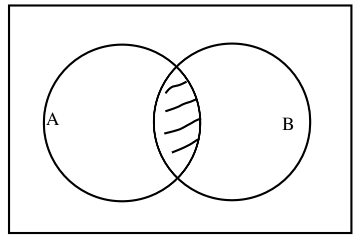
1.2 Venn Diagrams and Partitions

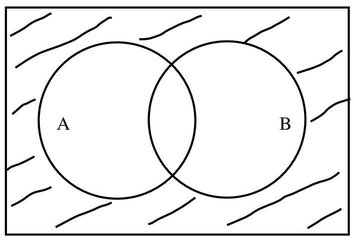
Venn Diagram of two sets



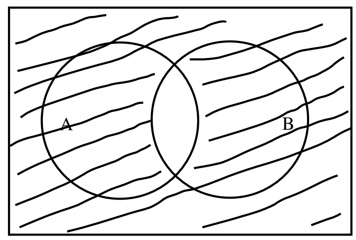
$A \cup B$



$A \cap B$



$A' \cap B'$



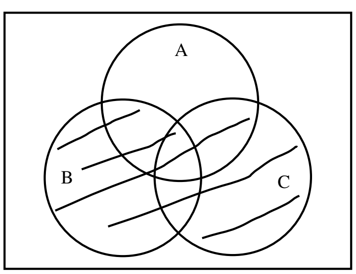
$A' \cup B'$

Formula: For any subsets A and B of a universal subset U

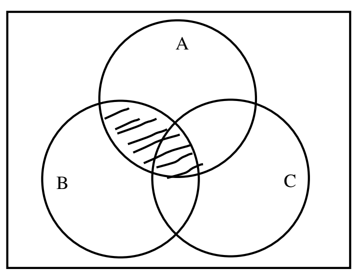
$$(A \cup B)' = A' \cap B'$$

$$(A \cap B)' = A' \cup B'$$

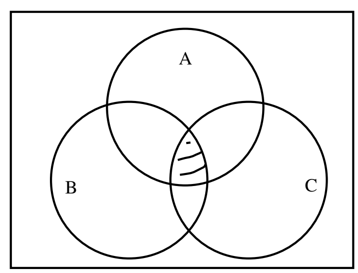
Venn Diagram of three sets



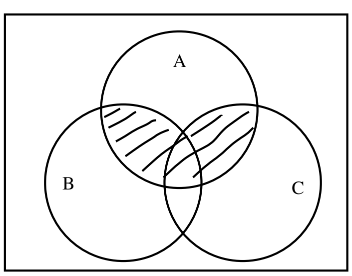
$B \cup C$



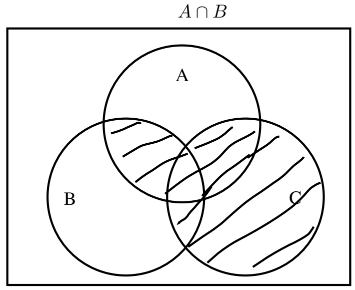
$A \cap B$



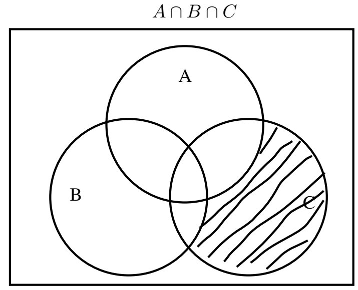
$A \cap B \cap C$



$A \cap (B \cup C)$



$(A \cap B) \cup C$



$A' \cap B' \cap C$

Partition: A "partition" of a set X is a collection of nonempty subsets of X which are pairwise disjoint and whose union is the entire set X.

Ex: Let $X = \{1, 2, 3, 4\}$ then

1. $\{1, 2\}$, $\{3\}$ and $\{4\}$ is a partition of X.
2. $\{1, 3\}$ and $\{2, 4\}$ is also a partition of X.
3. $\{1, 3\}$ and $\{2, 3, 4\}$ is NOT a partition of X because the sets not pariwise disjoint.
4. $\{2, 3\}$ and $\{4\}$ is NOT a partition of X because the union is not X.

Definition: Let A be a set. The number of elements in A is denoted $n(A)$.

Ex: If $A = \{a, b, c\}$, then $n(A) = 3$.

Partition principle: If X is partitioned into subsets X_1, X_2,..., X_k then

$$n(X) = n(X_1) + n(X_2) + \cdots + n(X_k)$$

Ex: Let X be a set, let $X_1 = \{a, c\}$, $X_2 = \{b, d, e\}$, and $X_3 = \{f, g\}$ be a partition of X then

$$n(X) = n(X_1) + n(X_2) + n(X_3) = 2 + 3 + 2 = 7$$

Formula: If A and B are sets then $n(A \times B) = n(A) \cdot n(B)$.

Ex: Let $A = \{3, g, f\}$ and $B = \{u, v\}$ then

$$n(A \times B) = n(A) \cdot n(B) = 3 \cdot 2 = 6.$$

Formula: If X_1, X_2,..., X_n are sets then $n(X_1 \times X_2 \times \cdots \times X_n) = n(X_1) \cdot n(X_2) \cdot \cdots \cdot n(X_n)$.

Ex: Let $n(X_1) = 3$, $n(X_2) = 5$, and $n(B) = 2$, find $n(X_1 \times X_2 \times B)$.

$$n(X_1 \times X_2 \times B) = n(X_1) \cdot n(X_2) \cdot n(B) = 3 \cdot 5 \cdot 2 = 30.$$