1.3 Size of sets

Formula 1: Let A and B be two sets. Then

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Formula 2: Let A and B be two DISJOINT sets. Then

$$n(A \cup B) = n(A) + n(B)$$

Formula 3: Let U be a universal set and A a subset. Then

$$n(U) = n(A) + n(A')$$

Ex 1: Let U be a universal set with two non-disjoint subsets A and B. Find $n(A \cup B)$ when $n(U) = 10$, $n(A) = 7$, $n(B) = 5$ and $n(A \cap B)$=4.
ANSWER: $n(A \cup B) = 8$.

Ex 2: Let U be a universal set with two non-disjoint subsets A and B. Find $n(A \cap B)$ when $n(U) = 15$, $n(A') = 7$, $n(B) = 5$ and $n(A' \cap B')$=4.
ANSWER: $n(A \cap B) = 2$

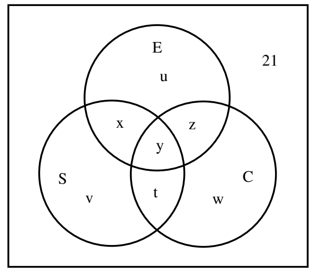
Ex 3: Let U be a universal set with two disjoint subsets A and B. Find $n(A \cap B')$ when $n(U) = 50$, $n(A) = 19$, and $n(B') = 32$.
ANSWER: $n(A \cap B') = 19$

Ex 4: Let U be a universal set with two non-disjoint subsets A and B. Find $n(A \cup B)$ when $n(U) = 110$, $n(B') = 48$, $n(A \cap B') = 23$ and $n(A \cap B)$=33.
ANSWER: $n(A \cup B) = 85$

Ex 5: Assume that 140 voters are asked their views: 66 favor raising the gas tax, 69 favor additional highway spending, and 36 favor both. How many of these voters favor neither the gas tax increase nor additional highway spending?
ANSWER: 41 voters favor neither.

Ex 6: Some automobiles are tested for production defects. The number of production defects is 30. 13 of them were major defects, 14 were design defects, and 8 were neither. How many of the design defects were major?
ANSWER: 6 design defects were major

Ex 7: Assume that 175 surveys are completed about a drug. Of those surveyed, 90 responded positively to effectiveness, 85 responded positively to side effects, and 86 responded positively to cost. Also, 43 responded positively to both effectiveness and side effects, 44 to effectiveness and cost, 42 to side effects and cost, and 21 to none of the items. How many of them responded to all three of them?
ANSWER: For this one we graph the Venn Diagram. We can write down 21 on the Venn Diagram but for the other numbers we will denote them x,y,z,u,v,w,t. We are looking for y.



E=effectiveness, S=side effects, C=cost

From the problem we have the following equations

$$
\begin{aligned}
&(1)\quad x + y = 43\\
&(2)\quad z + y = 44\\
&(3)\quad t + y = 42\\
&(4)\quad u + x + y + z = 90\\
&(5)\quad v + x + y + t = 85\\
&(6)\quad w + t + y + z = 86\\
&(7)\quad x + y + z + t + u + v + w + 21 = 175
\end{aligned}
$$

The goal is to compute y. First we add equations (4)+(5)+(6):

$$(8)\quad u + w + v + 2x + 2z + 2t + 3y = 261,$$

then we plug (7) into (8) and we obtain

$$(9)\quad x + z + t + 2y = 107.$$

Now we add (1)+(2)+(3)

$$(10)\quad x + z + t + 3y = 129.$$

Now we just plug (9) into (10) and we get

$$107 + y = 129 \Longrightarrow y = 22.$$

We conclude that 22 responded to all three.

<u>Ex 8:</u> Assume that 350 students are surveyed with the following results:

- 173 lives in the west
- 172 lives in a large city
- 187 are married
- 70 live in the west in a large city
- 90 are married and live in a large city
- 98 are married and live in the west
- 39 are married and live in the west in a large city.

How many are unmarried, do not live in a large city, and do not live in the west?
<u>ANSWER</u>: 37 are unmarried, do not live in a large city, and do not live in the west