1 Events

Definition: An "event" is a subset of a sample space.
Ex 1: A telephone sales representative makes successive calls to potential customers, and the result of each call is recorded as a sale (S) or a no sale (N). Calls are continued until either two sales are made (these calls need not be consecutive), two consecutive calls result in no sale , or a total of 4 calls is made.

1. What is the sample space?
2. Find the event E that exactly one sale is made.
3. Find the event F that exactly one call results in no sale.

Ex 2: The exhaust system of an automobile is to be given emission and vibration checks. The result of an emissions check can be very satisfactory (V), satisfactory (S), marginal (M), or unsatisfactory (U), and the result of a vibration check can be high vibration (H) or low vibration (L).

1. What is the sample space for this situation?
2. Find the event that the emissions check is satisfactory.

2 Probability of events

Definition: Let E be an event and let S be the sample space of an experiment. If each outcome in the sample space is EQUALLY LIKELY, then

$$Pr[E] = \frac{n(E)}{n(S)}$$

Remark: The probability of an event is a number between 0 and 1.

Definition: If all the outcomes from a sample space S are EQUALLY LIKELY then the probability of an outcome is

$$Pr[\text{one outcome}] = \frac{1}{n(S)}$$

Definition: Let E be an event then

$$Pr[E] + Pr[E'] = 1$$

Ex 3: There are balls numbered 1 through 8 in a box, and an experiment consists of randomly selecting 2 balls one after another without replacement.

1. How many outcomes does this experiment have?
2. What probability should be assigned to each outcome?
3. What probability should be assigned to the event that at least one ball has an odd number?

Ex 4: A product code consists of a digit from the set $\{8, 7, 1, 5, 3, 4\}$ followed by 4 letters, not necessarily distinct, selected from the set $\{K, T, E, A\}$. For example, 4KTET is such a code.

1. How many codes are possible?
2. What probability should be assigned to each code?

3. What probability should be assigned to the event that the code contains the number 4?

4. What probability should be assigned to the event that the code contains at least one letter K?

Ex 5: You weigh 350 duck hatchlings. You find that 83 are slightly underweight, 9 are severely underweight, and the rest are normal.

1. What probability should be assigned to a single duck hatchlings being slightly underweight?

2. What probability should be assigned to a single duck hatchlings being severely under- weight?

3. What probability should be assigned to a single duck hatchlings being normal?

3 Weighted probability

Definition: Suppose that an experiment has a sample space S with n outcomes O_1, O_2,...,O_n. We denote by w_1,...,w_n the probability of each outcome O_1,...,O_n. We have the following:

$$Pr[O_i] = w_i$$
$$0 \leq w_i \leq 1 \text{ for } i = 1, \ldots, n$$
$$w_1 + w_2 + \cdots + w_n = 1$$

Definition: Let E be an event such that $E = \{O_1, O_2, O_5\}$. Then

$$Pr[E] = Pr[O_1] + Pr[O_2] + Pr[O_5] = w_1 + w_2 + w_3$$

Ex 6: In a finite mathematics class with 300 students, 9% withdraw, 12% receive an A, 17% receive a B, 39% receive a C, 15 % receive a D, and 8% receive an F.

1. What probability should be assigned to the event pass the course?

2. What probability should be assigned to the event withdraw or fail the course?

Ex 7: A die is weighted so that the odd numbers are equally likely to come up, all the even numbers are equally likely to come up, and the odd numbers are 2 times as likely to come up as the even numbers.

1. What is w_1?

2. What is w_6?

3. What is w_2?

Ex 8: A die is weighted so that 4, 5, 3, and 6 are equally likely, a 2 is twice as likely as a 6, and a 1 is four times as likely as a 2.

1. What is w_4?

2. What is w_2?

3. What is w_1?

Ex 9: Let $S = \{O_1, O_2, O_3, O_4, O_5, O_6\}$ and suppose that $w_1 = 0.21$, $w_2 = 0.28$, $w_3 = 0.15$, $w_4 = 0.05$, $w_5 = 0.14$, $w_6 = 0.17$. Let $E = \{O_5, O_2\}$ and $F = \{O_4, O_6\}$.

1. Find $Pr[E]$.

2. Find $Pr[F]$.