

2.2/2.3 Counting

Suppose an experiment consists of choosing k objects from a total of n objects. To figure out how many different outcomes are possible, we need to answer two questions:

1. **Is replacement allowed?**
2. **Does order matter?**

Once these questions have been answered, use the following table:

Is replacement allowed?	Does order matter?	Number of outcomes
YES	YES	n^k
YES	NO	-
NO	YES	$P(n, k)$
NO	NO	$C(n, k)$

Factorial: $n! = n \cdot (n - 1) \cdot (n - 2) \cdot \dots \cdot 1$.

Multiplication principle:

$$n^k = \underbrace{n \cdot n \cdot \dots \cdot n}_{k \text{ times}}$$

Permutation:

$$P(n, k) = \frac{n!}{(n - k)!} = \underbrace{n \cdot (n - 1) \cdot \dots \cdot (n - k + 1)}_{k \text{ factors}}$$

Combination:

$$C(n, k) = \frac{n!}{k!(n - k)!} = \frac{P(n, k)}{k!}$$

Ex 1: Assume that you have a committee of 13 members, and that you must choose a president, a vice president and a secretary. How many ways can the selection be made?

Ex 2: You are ordering a 3-topping pizza. The topping choices are mushrooms, anchovy, tofu, pineapple, and green pepper. How many different types of pizza are possible?

Ex 3: Let $X = \{L, M, N, O, P, Q\}$. A code consists of four (not necessarily distinct) letters from the set X . How many codes are possible?

Ex 4: Let $X = \{L, M, N, O, P, Q\}$. A code consists of four distinct letters from the set X . How many codes are possible?

Ex 5: Assume that 12 students, including John, are working on a project report, which contains an introduction, four chapters, and a conclusion for a total of six sections.

1. In how many ways can the students divide up the report writing if each student writes a different section?
2. In how many ways can the assignments be made if John writes the introduction?

Ex 6: Assume that a committee consists of 15 members including Cecil. There are 3 offices to be filled: chairperson, vice-chairperson, and secretary.

1. In how many different ways can the offices be filled if each person can hold at most one office?
2. In how many of these ways is Cecil the chairperson?
3. In how many of these ways is Cecil an officer?

Ex 7: There is a casting for a play in a community theater. Assume that there are 6 unfilled roles: 2 male and 4 female. There are 4 men and 5 women, including Jennifer, auditioning for a part in the play.

1. How many different casts are there?
2. How many of these casts include Jennifer?

Ex 8: The producer of a variety television show wants to start with a commercial, followed by 2 skits, and finished with one commercial. Assume that the producer has 7 skits and 6 commercials available. We also assume that a skit cannot be repeated but commercials can be repeated. In how many ways can the producer create the television show?

Ex 9: An urn contains 9 red balls and 6 blue balls. An experiment consists of drawing four balls simultaneously from the urn. How many outcomes contain at least one ball of each color?

Ex 10: Assume that a committee consists of 7 Republicans and 4 Democrats. A subcommittee consisting of 6 people is to be selected.

1. How many such subcommittees are possible if each subcommittee must contain exactly 4 Republicans and 2 Democrats?
2. How many such subcommittees are possible if each subcommittee must contain at least 1 but no more than 2 Democrats?

Ex 11: Assume that you have 8 dimes and 3 quarters (all distinct), and you select 4 coins.

1. In how many ways can the selection be made?
2. In how many ways can the selection be made if all the coins are dimes?
3. In how many ways can the selection be made if you select 2 dimes and 2 quarters?
4. In how many ways can the selection be made so that at least 3 coins are dimes?