

D116: 3.3 Stochastic Processes and Trees

An experiment which consists of a sequence of subexperiments is called a *stochastic process*.

Ex 1: You have two boxes a and b . Box a contains 3 red balls and 2 green balls, box b contains 4 red balls and 3 green balls. The experiment consists of two steps, first selecting a box at random and then selecting a ball. What is the probability of selecting a red ball?

Ex 2: Assume that you have an urn with 4 red balls and 6 green balls from which you randomly select two balls without replacement.

1. What is the probability that exactly one green ball is chosen?
2. What is the probability that the second ball is green given that at least one of the balls is green?

Ex 3: Assume that a business makes a profit with probability 0.5 in the first year. For each year thereafter, the business makes a profit with probability 0.6 if it made a profit in the previous year, and with probability 0.3 if it did not make a profit in the previous year. What is the probability that the business makes a profit in exactly two of its first three years?

Ex 4: Assume that you have two bags a and b and each contains 6 balls. Bag a contains 2 red and 4 white, while bag b contains 3 red, 2 white, and 1 blue. You randomly select one ball from bag a , note the color, and place the ball in bag b . You then select a ball from bag b at random and make note of its color.

1. What is the probability that both balls are red?
2. What is the probability that both balls are red given that the first ball you drew was red?

Ex 5: You want to find the missing probabilities, the first outcomes is one of (A, B) and the second outcome is one of $(1, 2)$. Assume that

- $Pr[2|A] = 3/5$
- $Pr[A2] = 1/5$
- $Pr[B1] = 5/9$
- $Pr[2|B] = 1/6$

Find $Pr[A]$, $Pr[1|A]$, $Pr[B]$, $Pr[1|B]$.

Bonus: Suppose that you have $Pr[A|B] = 1/4$, $Pr[A] = 3/8$, and $Pr[B] = 1/2$.

1. What is $Pr[B|A]$?
2. What is $Pr[B|A']$?