

D116: 3.5 Bernoulli Trials

Bernoulli Trials: probability of obtaining exactly r success (and therefore $n - r$ failures) in a Bernoulli process consisting of n trials with success probability p and failure probability $q = 1 - p$ is

$$Pr[r \text{ successes}] = C(n, r)p^r q^{n-r}$$

for $r = 0, 1, 2, \dots, n$.

Some Formulas: In any Bernoulli process with n trials you have

$$Pr[0 \text{ successes}] + Pr[1 \text{ successes}] + \dots + Pr[n \text{ successes}] = 1.$$

So for examples we have:

$$Pr[\text{at least 1 successes}] = 1 - Pr[0 \text{ successes}]$$

and

$$Pr[\text{at least 2 successes}] = 1 - Pr[0 \text{ successes}] - Pr[1 \text{ successes}]$$

Ex 1: For a Bernoulli trials compute the following probabilities:

- 1 success in 3 trials with $p = .4$
- 3 successes in 5 trials with $p = \frac{1}{3}$
- 2 successes in 5 trials with $q = 0.6$
- 3 failures in 5 trials with $p = \frac{2}{3}$.
- At least 2 successes in 6 trials with $p = 0.55$
- At least 4 failures in 8 trials with $p = 0.8$

Ex 2: For a Bernoulli trials with $n = 7$ and $p = .67$. Find the probability of at least 3 successes and at least 1 failure.

Ex 3: Assume that the coin is weighted so that a tail is 3 times as likely as a head. The coin is flipped 12 times. What is the probability that both heads and tails occur?

Ex 4: A high school basketball player makes one-third of his three points shots. If we assume that his shots are Bernoulli trials, how many must he shoot to have a probability of at least .7 of making at least 1 of them.

Ex 5: A professor who intends to bring her briefcase to the office each morning forgets it one-sixth of the time. Assume that her forgetting is a Bernoulli process.

1. What is the probability that she remembers to bring her briefcase every day in one week (5 days)?
2. What is the probability that she forgets to bring her briefcase every day in one week (5 days)?
3. What is the probability that she forgets to bring her briefcase at least one day in one week (5 days)?