

D116: 4.1 Random variables and probability density functions

Random variable: It is a number associated to outcomes.

Density function: It is the probability of each numbers that the random variables can take.

For example, let us say you flip a coin once. You win \$1 if it is heads, and loose \$1 if it is tails. The random variable X will be your gain or loss. The values of X and its density function can be written as follows:

Value of X	probability
1	$Pr[X = 1] = \frac{1}{2}$
-1	$Pr[X = -1] = \frac{1}{2}$

Ex 1: An unfair coin with $Pr[H] = .4$ is flipped 3 times. A random variable X is defined as the number of heads plus twice the number of tails. Find the values of X and its density function.

Ex 2: A student committee has 6 members, 4 females and 2 males. Two students are selected at random, and a random variable X is defined to be the number of females selected. Find the density function of X .

Ex 3: A vending machine yields the item selected 80 percent of the time and no item at all 20 percent of the time. Three individuals attempts to use the machine. Let the random variable X as the number of individuals who obtain the item selected. Find the values taken by X and the density function of X , under the assumption that the attempts to use the machine from a Bernoulli process.

Ex 4: An experiment consists of randomly drawing 3 cards from a standard deck (without replacement). The random variable Z is defined to be 5 times the number of aces plus 2 times the number of face cards which are not aces. Find the density function of Z .

Ex 5: In a box containing 12 hand calculators, 9 works and 3 do not work. A sample of 5 calculators is selected at random from the box. A random variable is defined which assigns to each outcomes the number of calculators which do not work. Find the values assumed by this random variables and its density function.

Ex 6: A student has 3 volumes of short stories and 2 novels on a bookshelf. She selects 3 books at random to take home over vacation, A random variables X is defined to be the number of novels selected. Find the values assumed by X and its density function.

Ex 7: There are 20 balls labeled 1 through 20 in a box. A ball with an even number is worth \$1, and a ball with an odd number is worth \$2. Three balls are selected simultaneously and at random. A random variable X is defined to be the total value in dollar of the 3 balls selected. Find the density function of X .

Ex 8: An unfair coin is weighted so that $Pr[h] = \frac{2}{3}$. The coin is flipped until a head appears or 4 consecutive tails appear. The random variables X is defined to be the total number of flips of the coin. Find the values assumed by X and the density function of X .

Ex 9: An experiment consists of rolling a fair die until the sum of all numbers which have appear exceeds 3. A random variable X is defined to be the total number of rolls. Find the density function of X .

Ex 10: You have two coins: one is fair and the other is unfair with $Pr[H] = \frac{2}{3}$. An experiment consists of selecting a coin at random and flipping it twice, noting the result of each flip. Define a random variable X to be the number of heads. Find the density function of X .

Ex 11: An experiment consists of randomly choosing two 4-digit numbers from the set of consecutive integers from 0000 to 9999. The selection is with replacement and we assume the selections are independent. Thus, possible selections are 0135 and 2553. If the two numbers selected are the same, you win \$500; and if they are not the same, you lose \$1. Find the density function of the random variable which gives you gain or loss.