

## D116: 4.2 Expected value

**Expected value:** Let  $X$  be a random variable that take the values  $x_1, x_2, \dots, x_k$ . The expected value of  $X$ , denoted  $E[X]$  is

$$E[X] = x_1Pr[X = x_1] + x_2Pr[X = x_2] + \dots + x_kPr[X = x_k]$$

**Expected value of a Bernoulli process:** If the random variable  $X$  follows a Bernoulli process consisting of  $n$  trials with success probability  $p$ , then

$$E[X] = n \cdot p.$$

**Example:** let us say you flip a coin once. You win \$1 if it is heads, and loose \$1 if it is tails. The random variable  $X$  will be your gain or loss. The values of  $X$  and its density function can be written as follows:

Value of $X$	probability
1	$Pr[X = 1] = \frac{1}{2}$
-1	$Pr[X = -1] = \frac{1}{2}$

Then the expected value of  $X$  is

$$E[X] = 1 \cdot \frac{1}{2} + (-1) \cdot \frac{1}{2} = 0.$$

**Sum of random variables:** If you have a random variable that is defined as the sum of two random variables,  $Z = X + Y$ , then

$$E[Z] = E[X] + E[Y].$$

Ex 1: Let  $X$  be a random variable with the following density function

Value of $X$	probability
-4	0.1
0	0.1
1	0.3
2	0.5

What is the expected value of  $X$ ?

Ex 2: Assume that a drawer contains 8 coins: 3 quarters, 3 dimes, and 2 nickels. One coin is randomly selected. What is the expected value of the coin selected (in cents)?

Ex 3: A fair die is rolled 12 times and the number that lands uppermost is noted. An outcome is called “high” if it is a 5 or a 6, and a random variable  $X$  is defined as the number of high outcomes in the 12 trials. Find  $E[X]$ .

Ex 4: You have two coins; one is fair and one is unfair with  $Pr[H] = \frac{2}{3}$ . An experiment consists of selecting a coin at random and flipping it twice, noting the results of each flip. Find the expected number of heads.

Ex 5: A batter hits with probability 0.25 against left-handed pitchers, and with probability 0.15 against right-handed pitchers. One month the player bats 22 times against left-handed pitchers and 61 times against right-handed pitchers. How many hits should the batter expect to get during this month?

Ex 6: A coin is weighted so that  $Pr[H] = \frac{2}{5}$ . The coin is flipped 10 times. A random variable  $X$  is defined by assigning to each outcome the number of heads obtained. Find  $E[X]$ .

Ex 7: A coin is weighted so that  $Pr[H] = \frac{2}{5}$ . The coin is flipped 3 times. A random variable  $X$  is defined by the twice number of heads minus the number of tails. Find  $E[X]$ .

Ex 8: A golfer makes 60 percent of her putts when they are less than 10 feet and 30 percent when they are 10 feet or more length. In one tournament she takes 32 putts from less than 10 feet and 78 from 10 feet or more. What is the expected number of putts made?

Ex 9: At a local carnival a game can be played with a fishpond containing 100 fish; 90 are white, 9 are red, and 1 is blue. A contestant randomly catches a fish and receives payment as follows:

- White: \$.30
  
- Red: \$1.00
  
- Blue: \$10.00

If it costs \$.60 to play this game, how much (on the average) does the carnival gain on each play?

Ex 10: An examination consists of 2 true-false questions and 2 multiple choice questions (5 options per questions), each with exactly 1 correct answer. If a student selects answers at random, one answer per each question, what is the expected number of correct answers?