

## D117: 5.1 Linear Equations

**Definition 1.** *General Form*

$$Ax + By = C.$$

**Definition 2.** *Slope between  $(x_1, y_1)$  and  $(x_2, y_2)$ :*

- if  $x_1 \neq x_2$ ,  $m = \frac{y_2 - y_1}{x_2 - x_1}$ .
- if  $x_1 = x_2$  then it is a vertical line and the slope is undefined.

**Ex:** Slope of the line going through  $(2, 1)$  and  $(3, 4)$ ,

$$m = \frac{4 - 1}{3 - 2} = 2$$

**Ex:** Slope of the line going through  $(-1, 1)$  and  $(-1, 3)$ , since  $x_1 = x_2 = -1$  the slope is undefined.

**Definition 3.** *Two lines are parallel if they have the same slope  $m$ .*

**Definition 4.** *Slope Form*

$$y = mx + b$$

where  $m$  is the slope and  $(0, b)$  is the  $y$ -intercept.

**Ex:** Find the slope of  $5x - 2y = 1$ .

You write the equation in the slope form

$$5x - 2y = 1 \implies -2y = -5x + 1 \implies y = \frac{5}{2}x - \frac{1}{2}.$$

The slope is  $\frac{5}{2}$ .

**Special cases:**

1.  $x = a$ , it is a vertical line with slope undefined and no  $y$ -intercept.
2.  $y = b$ , it is a horizontal line with slope  $m = 0$  and  $(0, b)$   $y$ -intercept.

**Definition 5.** *Intercepts*

- $x$ -intercept is when  $y = 0$
- $y$ -intercept is when  $x = 0$ .

**Ex:** Find the  $x$  and  $y$  intercept for  $y = 2x + 1$ .

1.  $x$ -intercept, plug  $y = 0$  and you have  $0 = 2x + 1 \implies x = -1/2$ , so the  $x$ -intercept is  $(-1/2, 0)$ .
2.  $y$ -intercept, plug  $x = 0$  and you have  $y = 2 \cdot 0 + 1 \implies y = 1$ , so the  $y$ -intercept is  $(0, 1)$ .

**Method to find the equation of a line:**

The equation of a line is  $y = mx + b$ , so you have to find  $m$  and  $b$ .

1. First you find the slope  $m$ .
2. Second you plug a point that belongs to the line in the equation  $y = mx + b$  to find the value for  $b$ .
3. Write the equation of the line.

**Ex:** Find the equation of the line going through  $(-1, 3)$  and  $(2, 1)$ .

1. First you find the slope:  $m = \frac{1 - 3}{2 - (-1)} = -\frac{2}{3}$ . So  $y = -\frac{2}{3}x + b$
2. Second you find  $b$  by plug in in one of the point, let's take  $(-1, 3)$ ,

$$3 = -\frac{2}{3}(-1) + b \implies 3 = \frac{2}{3} + b \implies b = 3 - \frac{2}{3} = \frac{9}{3} - \frac{2}{3} = \frac{7}{3}.$$

3. The answer is  $y = -\frac{2}{3}x + \frac{7}{3}$

**Ex:** Equations of the line going parallel to line going through  $(1, 4)$  and  $(2, 6)$ , and passes through  $(1, 2)$ .

1. It has the same slope as the line going through  $(1, 4)$  and  $(2, 6)$ ,

$$m = \frac{6 - 4}{2 - 1} = 2,$$

so  $y = 2x + b$ .

2. Now we compute  $b$  by plugging one point, we know that  $(1, 2)$  satisfies the equation:

$$2 = 2 \cdot 1 + b \implies b = 2 - 2 = 0$$

3. The equation of the line is  $y = 2x$ .

**Ex:** Find the equation of the line that is parallel to  $2y + 2x = 1$  and has y-intercept 3.

1. First you find the slope. Since the line is parallel to  $2y + 2x = 1$ , it has the same slope. You write  $2y + 2x = 1$  in the slope form

$$2y + 2x = 1 \implies 2y = -2x + 1 \implies y = -x + \frac{1}{2},$$

thus  $m = 1$

2. Second you find  $b$  by plug in in one of the point. The y-intercept is 3 which means that  $(0, 3)$  belongs to the line.

$$3 = 1 \cdot 0 + b \implies b = 3$$

3. The answer is  $y = x + 3$