D117: 5.2 System of Linear Equations in Two Variables

Substitution method:

$$\begin{cases} 2x - 5y = -16, \\ 3x + y = 10. \end{cases}$$

1. *Pick one equation and solve it for one variable.* Since y has no coefficient in the second equation, we are going to solve the second equation for y:

$$3x + y = 10 \Longrightarrow y = 10 - 3x$$

2. *Plug this equation in the other equation,*

$$2x - 5(10 - 3x) = -16 \Longrightarrow 2x + 15x - 50 = -16 \Longrightarrow 17x = 34 \Longrightarrow x = 2.$$

3. Once we have the value of one variable, in this case x, we substitute x by its value,

$$y = 10 - 3x = 10 - 3 \cdot 2 = 10 - 6 = 4.$$

4. The solution is $x = 2, y = 4$ or $(2, 4)$.

Addition method:

$$\begin{cases} 4x + 2y = 3, \\ 7x + 3y = -2. \end{cases}$$

1. *You multiply one or both equation to cancel one variable.* Let's multiply the first one by -3 and the second one by 2:

$$\begin{cases} -12x - 6y = -9, \\ 14x + 6y = -4. \end{cases}$$

   We have $-3y$ in the first one and $3y$ in the second, when we add them the y variable will disappear.

2. *Add the two equations:*

$$\begin{array}{r} -12x - 6y = -9 \\ 14x + 6y = -4 \\ \hline 2x = -13 \end{array}$$

   Therefore $x = -\dfrac{13}{2}$

3. Once we have the value of one variable, in this case x, we substitute x by its value in one of the equations,

$$-12(-\frac{13}{2}) - 6y = -9 \Longrightarrow y = \frac{29}{2}$$

4. The solution is $x = -\dfrac{13}{2}, y = \dfrac{29}{2}$ or $(-\dfrac{13}{2}, \dfrac{29}{2})$.

No solution:

$$\begin{cases} 4x - y = 3, \\ 8x - 2y = 2. \end{cases}$$

From the first equation we have,

$$y = 4x - 3.$$

We plug this equation in the second equation,

$$8x - 2(4x - 3) = 2 \Longrightarrow 8x - 8x + 6 = 2 \Longrightarrow 6 = 2.$$

Since $-6 = 2$ is NOT TRUE, there are no solutions.

When you solve by substitution or addition a system of linear equations, if at some point you obtain an equation, without any x or y, that is WRONG (like $0 = -1$ or $15 = 2$), then there are NO SOLUTIONS.

Infinitely many solution:

$$\begin{cases} 4x + 2y = 2, \\ 6x + 3y = 3. \end{cases}$$

We multiply the first one by -3 and the second one by 2,

$$\begin{cases} -12x - 6y = -6, \\ 12x + 6y = 6. \end{cases}$$

When we add both line we get

$$0 = 0.$$

Since $0 = 0$ is ALWAYS TRUE, there are infinitely many solutions. This means the two equations are actually the same and therefore only one of them matters.

Solve the first equation for x and you obtain,

$$x = -\frac{1}{2}y + \frac{1}{2},$$

so one way to write the solutions is $(-\frac{1}{2}y + \frac{1}{2}, y)$.

$$y = -2x + 1,$$

so another way to write the solutions is $(x, -2x + 1)$.

When you solve by substitution or addition a system of linear equations, if at some point you obtain an equation, without any x or y, that is always TRUE (like $0 = 0$ or $2 = 2$), then there are INFINITELY MANY SOLUTIONS.