D117: 6.1 Matrix Algebra

Adding Matrices
You can only add matrices of the same dimension. To add then, you just add each coefficients.

$$\begin{pmatrix} 1 & 9 \\ 5 & 6 \end{pmatrix} + \begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 8 \\ 8 & 6 \end{pmatrix}$$

Multiplying by a scalar (real number)
If A is a mtrix and u is a real number then to compute uA you multiply each coefficient of A by u.

$$-2\begin{pmatrix} 1 & 9 \\ 5 & 6 \end{pmatrix} = \begin{pmatrix} -2 & -18 \\ -10 & -12 \end{pmatrix}$$

Row-column multiplication

$u = [u_1 \quad u_2 \quad \cdots \quad u_n]$ a row vector of dimension n and $x = \begin{pmatrix} x_1 \\ x_1 \\ \vdots \\ x_n \end{pmatrix}$ a column vector of dimension n,

$$ux = u_1x_1 + u_2x_2 + \cdots + u_nx_n$$

Multiplying Matrices
Let A of dimension $m \times n$ and B of dimension $n \times k$ then AB is of dimension $m \times k$.

$$AB = C \Longleftrightarrow c_{ij} = [i^{th} \quad row \quad of \quad A] \times [j^{th} \quad column \quad of \quad B]$$

$$\begin{pmatrix} 1 & 9 \\ 5 & 6 \end{pmatrix} \times \begin{pmatrix} 2 & -1 & 4 \\ 3 & 0 & -2 \end{pmatrix} = \begin{pmatrix} 1\cdot 2 + 9\cdot 3 & 1\cdot(-1) + 9\cdot 0 & 1\cdot 4 + 9\cdot(-2) \\ 5\cdot 2 + 6\cdot 3 & 5\cdot(-1) + 6\cdot 0 & 5\cdot 4 + 6\cdot(-2) \end{pmatrix} = \begin{pmatrix} 23 & -1 & -14 \\ 28 & -5 & 8 \end{pmatrix}$$

Careful: AB is not the same as BA.

Powers of matrices

$$A^2 = A \times A, \;\; A^3 = A^2 \times A = A \times A \times A, \;\; \cdots$$