D117: 7.3 Graphical Solution of Linear Programming Problems with Two Variables

Feasible set = the set representing all the constraints.

Different types of feasible sets

1. Empty sets
2. Bounded sets
3. Unbounded sets

Theorem on Solutions of Linear Programming Problems
Suppose that the feasible set is not empty and let f be the objective function.

1. if the feasible set is <u>bounded</u>, then f attains its maximum and minimum at a corner point of the feasible set.
2. if the feasible set is <u>unbounded</u> and has at least one corner point, then exatcly one of the following holds:
   (a) f attains its maximum/minimum at a corner point of the feasible sets.
   (b) f takes arbitrarily large positive/negative values on the feasible set.

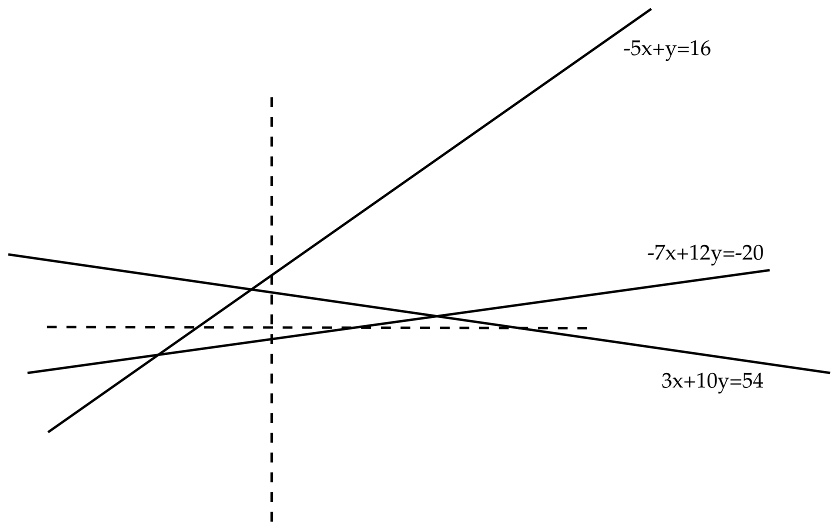
Solution Method for Linear Programming Problems

1. Graph the feasible set for the problem.
2. Determine the coordinates of each corner point.
3. Select the largest (if asked to maximize) or smallest (if asked to minimize) value.
   (a) if the feasible set is <u>bounded</u>, the value selected is the answer.
   (b) if the feasible set is <u>unbounded</u>: Plug each corner point into the objective and pick the one that gives the mininal (if you looking for minimum) or maximal (if you are looking for maximum) value. LEt call P this corner point.
      - If P lies at the intersection of two line segments, it is the maximum (resp. minimum) of the objective function subject to the given constraints.
      - If P lies at the intersection of a ray and a line segment, choose a point on the ray and plug it in to the objective function. If the value is larger (resp. smaller) than the value obtained when plugging P into the objective function, then the objective function does not have a maximum (resp. minimum) subject to the given constraints. Otherwise, the value obtained by plugging P into the objective function is the maximum (resp. minimum) of the objective function subject to the given constraints.
      - If P lies at the intersection of two rays, choose a point on each ray and plug them in to the objective function. If either value is larger (resp. smaller) than the value obtained when plugging P into the objective function, then the objective function does not have a maximum (resp. minimum) subject to the given constraints. Otherwise, the value obtained by plugging P into the objective function is the maximum (resp. minimum) of the objective function subject to the given constraints.

Example 1: Find the maximum value of the objective function $7x - 14y$, subject to the constraints

$$\begin{aligned} &3x + 10y \leq 54 \\ &-5x + y \leq 16 \\ &-7x + 12y \geq -20 \end{aligned}$$

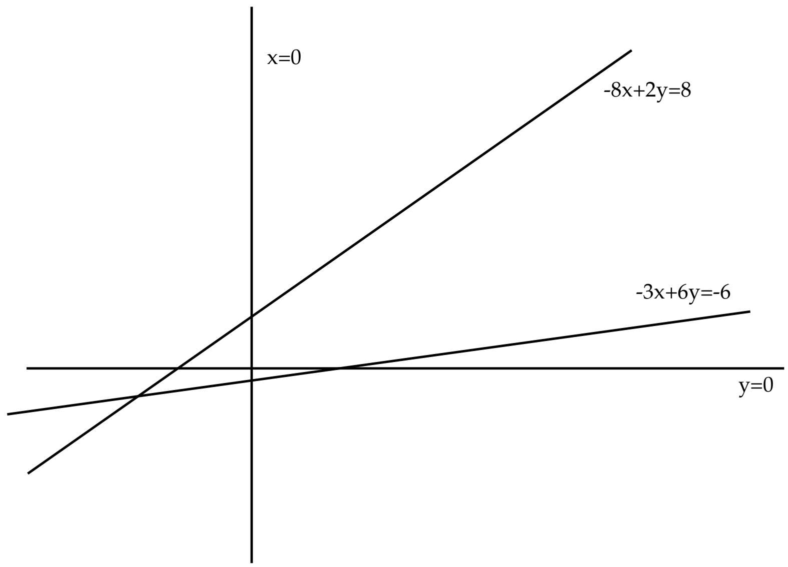
Find the maximum value of the ojective function $-5x + 4y$ and mimimum value of the objective function $-7x + 17y$ subject to the same constraints.



Example 2: Find the minimum value of the objective function $3x + 5y$, subject to the constraints

$$\begin{aligned} &x \geq 0 \\ &y \geq 0 \\ &-8x + 2y \leq 8 \\ &-3x + 6y \geq -6 \end{aligned}$$

Find the maximum value of the ojective function $5x + \frac{1}{2}y$ and mimimum value of the objective function $-x - 7y$ subject to the same constraints.



Example 3: Find the maximum value of the objective function $7x - 4y$, subject to the constraints

$$y \geq -x$$
$$2y + x \leq 2$$
$$2y - x \leq -6$$

Find the maximum value of the ojective function $-3x + 4y$ and mimimum value of the objective function $7x - 2y$ subject to the same constraints.