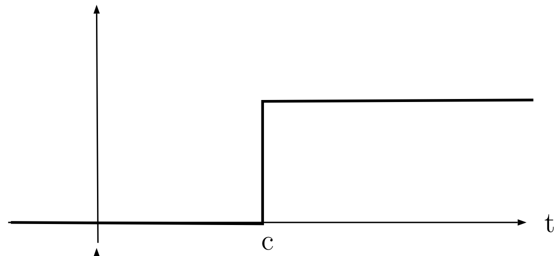
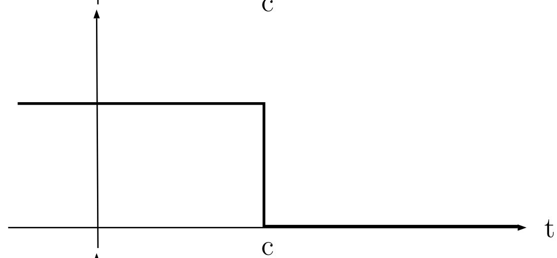
M250 Notes on Laplace Transform part II

Step Functions

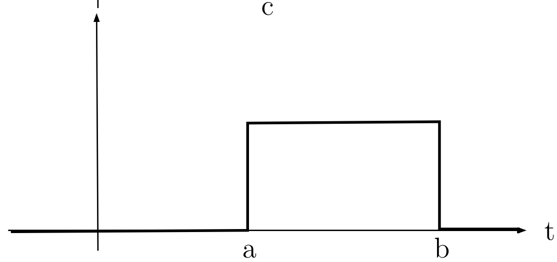
$$u_c(t) = \begin{cases} 0, & t < c \\ 1, & t \geq c \end{cases}$$



$$1 - u_c(t) = \begin{cases} 1, & t < c \\ 0, & t \geq c \end{cases}$$



$$u_a(t) - u_b(t) = \begin{cases} 0, & t < a \\ 1, & a \leq t < b \\ 0, & t \geq b \end{cases}$$



Natations

In the book they define

$$\mathcal{U}(t-c) = u_c(t)$$

Laplace transform of step functions

$$\mathcal{L}\left\{u_c(t)\right\} = \frac{e^{-cs}}{s}, \qquad \mathcal{L}\left\{u_c(t)f(t-c)\right\} = e^{-cs}\mathcal{L}\left\{f(t)\right\}$$

Pulse Function (Dirac)

$$\delta(t) = 0, \text{ if } t \neq 0 \text{ and } \int_{-\infty}^{\infty} \delta(t) dt = 1$$

$$\delta(t-c) = 0, \text{ if } t \neq c \text{ and } \int_{-\infty}^{\infty} \delta(t-c) dt = 1$$

Property.

$$\int_{-\infty}^{\infty} \delta(t-c) f(t) dt = f(c).$$

Laplace transform of Dirac function

$$\mathcal{L}\{\delta(t)\} = 1, \qquad \mathcal{L}\{\delta(t-c)\} = e^{-cs}, \qquad \mathcal{L}\{\delta(t-c)f(t)\} = e^{-cs} f(c).$$