M250 Notes on Autonomous system

Autonomous equations

Definition: An autonomous equation is of the form

$$y' = f(y). \tag{1}$$

Critical points: (or equilibrium points) Zeros of f

$$f(y) = 0. \tag{2}$$

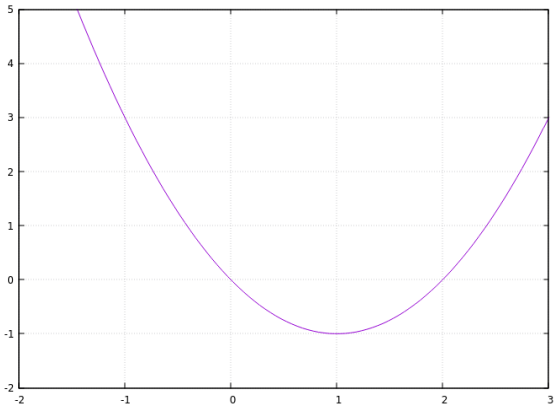
Stable equilibrium: You go towards the point.

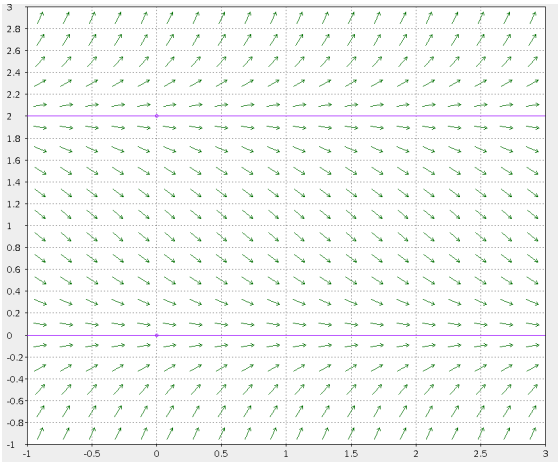
Unstable equilibrium: (asymptotically unstable) You go away from the point.

Semi-stable equilibrium: One side goes toward the point, the other side away from the point.

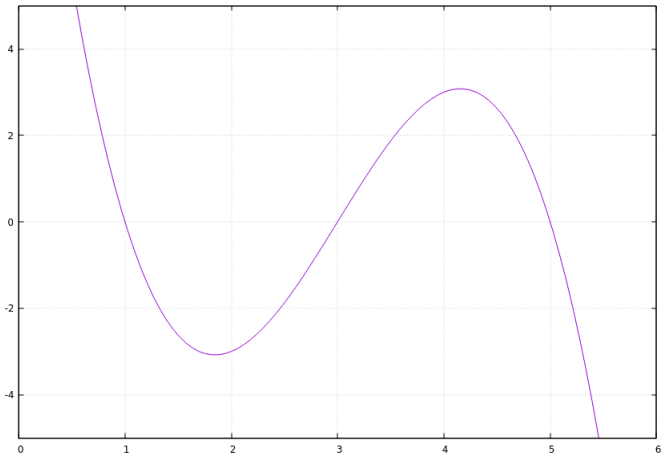
Example 1 $y' = y(y-2)$

1. What are the critical points?
2. Are they stable or unstable?
3. Behavior at $t \to \infty$.



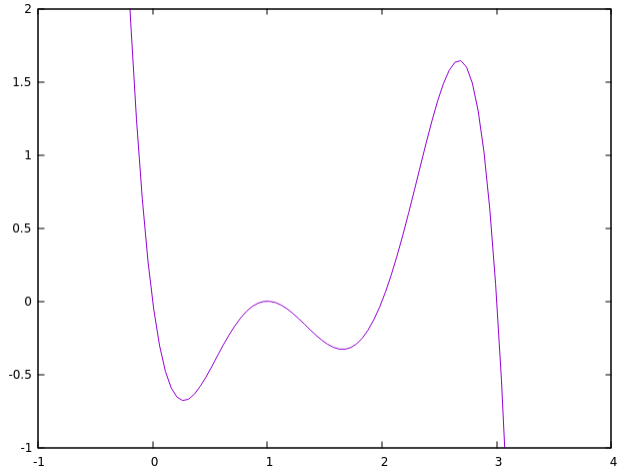


Example 2 $y' = f(y)$



1. What are the critical points?
2. Are they stable or unstable?
3. Sketch the solutions in the $y - t$ plane (direction field).
4. Behavior at $t \to \infty$.

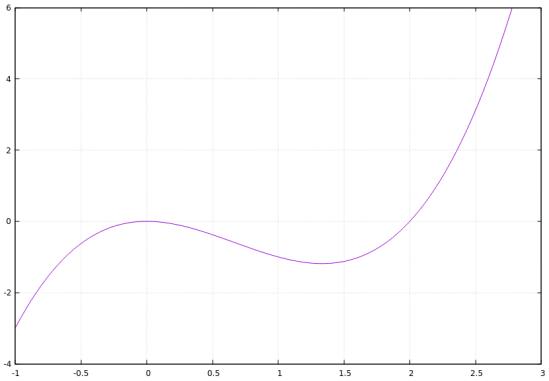
Example 3 $y' = f(y)$

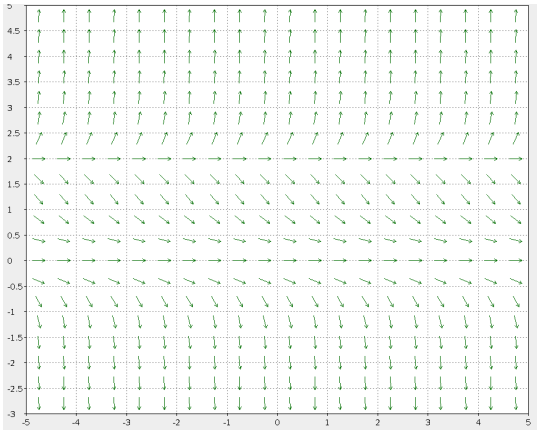


1. What are the critical points?
2. Are they stable or unstable?
3. Sketch the solutions in the $y - t$ plane (direction field).
4. Behavior at $t \to \infty$.

Example 5 $y' = y^3 - 2y^2$

1. What are the critical points?
2. Are they stable or unstable?
3. Behavior at $t \to \infty$.





Population dynamics

Model 1 Exponential growth: $y' = ry$

Model 2 Logistic equation: $y' = r(1 - \frac{y}{k})y$

Model 3 Logistic growth with threshold: $y' = -r\left(1 - \frac{y}{T}\right) \cdot \left(1 - \frac{y}{K}\right) y$, with $0 < T < K$.

Example : $y' = -1\left(1 - \frac{y}{5}\right) \cdot \left(1 - \frac{y}{10}\right) y$