M251 Notes on Two-points Boundary Value Problem

Homogeneous Dirichlet boundary conditions:

$$\begin{cases} y'' + \lambda y = 0, \;\; 0 < x < L \\ y(0) = 0, \;\; y(L) = 0. \end{cases}$$

The eigenvalues and eigenfunctions are

$$\lambda_n = \left(\frac{n\pi}{L}\right)^2, \;\; y_n(x) = \sin\frac{n\pi}{L}x, \;\; n = 1, 2, 3\ldots$$

Homogeneous Neumann boundary conditions:

$$\begin{cases} y'' + \lambda y = 0, \;\; 0 < x < L \\ y'(0) = 0, \;\; y'(L) = 0. \end{cases}$$

The eigenvalues and eigenfunctions are

$$\lambda_n = \left(\frac{n\pi}{L}\right)^2, \;\; y_n(x) = \cos\frac{n\pi}{L}x, \;\; n = 0, 1, 2\ldots$$

Homogeneous mixed boundary conditions 1:

$$\begin{cases} y'' + \lambda y = 0, \;\; 0 < x < L \\ y(0) = 0, \;\; y'(L) = 0. \end{cases}$$

The eigenvalues and eigenfunctions are

$$\lambda_n = \left(\frac{(2n-1)\pi}{2L}\right)^2, \;\; y_n(x) = \sin\frac{(2n-1)\pi}{2L}x, \;\; n = 1, 2, 3\ldots$$

Homogeneous mixed boundary conditions 2:

$$\begin{cases} y'' + \lambda y = 0, \;\; 0 < x < L \\ y'(0) = 0, \;\; y(L) = 0. \end{cases}$$

The eigenvalues and eigenfunctions are

$$\lambda_n = \left(\frac{(2n-1)\pi}{2L}\right)^2, \;\; y_n(x) = \cos\frac{(2n-1)\pi}{2L}x, \;\; n = 1, 2, 3\ldots$$